On the interconnection structures of irreversible physical systems

Eberard, Damien; Maschke, Bernhard; Schaft, Arjan J. van der

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On the Interconnection Structures of Irreversible Physical Systems

Damien Eberard[1], Bernhard Maschke[2], and Arjan J. van der Schaft[1]

[1] Institute for Mathematics and Computer Science, University of Groningen, the Netherlands
eberard@math.rug.nl
[2] Laboratoire d'Automatique et de Génie des Procédés UMR CNRS 5007, Université Lyon 1, France
maschke@lagep.univ-lyon1.fr

Summary. An energy balance equation with respect to a control contact system provides port outputs which are conjugated to inputs. These conjugate variables are used to define the composition of port contact systems in the framework of contact geometry. We then propose a power-conserving interconnection structure, which generalizes the interconnection by Dirac structures in the Hamiltonian formalism. Furthermore, the composed system is again a port contact system, as illustrated on the example of a gas-piston system undergoing some irreversible transformation.

1 Introduction

The properties of the control systems arising from models of physical systems reveal to be extremely useful for the design of control laws of nonlinear systems not only for stabilizing purposes but also for the design of the closed-loop behavior [19]. For electro-mechanical systems, the Lagrangian and Hamiltonian framework revealed to be best suited to represent their physical properties. Their state space is naturally endowed with symplectic, Poisson or Dirac structures arising from the variational formulation and eventual symmetries [1] [3] [11] or directly from the interconnection structure of complex physical systems like electrical or hydraulic networks and spatial mechanisms [12] and constraints [20] [21]. However the Hamiltonian and Lagrangian framework represent only the dynamics of reversible physical systems (in the sense of Thermodynamics). If the dissipation can no more be neglected, usually the Hamiltonian systems are augmented with a dissipative output feedback term [4] and lead to define dissipative Hamiltonian systems (defined on a so-called Leibniz bracket [15]) augmented with input and outputs maps [19]. From a thermodynamic perspective this means that the Hamiltonian function represent the free energy of the physical system which is not conserved. However there is a way of representing simultaneously (internal) energy conservation and irreversibility which uses model structures arising from Irreversible Thermodynamics and which has been developed in the context of Chemical Engineering. These systems are defined on the differentiable manifolds endowed with a contact structure which is canonically associated with the phase

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space in Reversible Thermodynamics [9] [3] [16]. On these contact manifolds, a class of control systems has been defined, called *conservative control contact systems*, which encompasses both reversible and irreversible systems and allows to express in both cases the conservation of the total energy [5] [6].

In this paper we shall consider port contact systems as defined in [7] and define their composition by power continuous interconnection structure which strictly generalize Dirac structures used for the composition of port Hamiltonian systems [21].

The sketch of the paper is the following. In the second section we shall briefly recall the definition and motivation of conservative contact systems. In the third section, we shall recall the definition of the port conjugated variables and then define the composition (or interconnection) of conservative contact systems. This is then illustrated in details on the example of a gas in a cylinder submitted to irreversible transformations.

2 Conservative Contact Systems

In this section we shall recall the motivation and definition of conservative contact systems. The reader is referred to [11] [3] and [1] for a detailed definition of the objects of contact geometry and will find a very brief summary in the appendix of this paper.

2.1 Definition and Motivation

The properties of thermodynamical systems are mathematically defined as the space of 1-jets of real functions of its extensive variables corresponding to their fundamental equation [8] [9] [13]. If one denotes the manifold of extensive variables (excluding the internal energy) by $\mathcal{N}$, it is known that the space of 1-jets of functions on $\mathcal{N}$ may be identified with $\mathbb{R} \times T^*\mathcal{N}$ [11]. This space is called the Thermodynamic Phase Space and has a canonical geometric structure, called contact form, which plays an analogous role as the Liouville form for cotangent bundle of differentiable manifolds. It may then be shown that the Thermodynamic Properties of physical systems (including thermodynamical systems but also mechanical and electro-magnetic systems) is defined by a Legendre submanifold of the associated Thermodynamic Phase Space $\mathcal{T}$ [9] [3] [13] [14].

It has been shown in previous work that the dynamics of *open* physical systems undergoing reversible or irreversible transformations may also be formulated in terms of contact geometry [5] [7] [6]. This has lead to the definition of a class of control contact systems that we have called conservative control contact systems [5] [6].

Definition 1. *A* conservative control contact system *is defined by the $m+5$-tuple $(\mathcal{M}, \mathcal{E}, \mathcal{L}, K_0, \ldots, K_m, \mathcal{U})$, where $(\mathcal{M}, \mathcal{E})$ is a strict contact manifold with $\mathcal{L}$ a given Legendre submanifold, the K_i's are smooth real-valued functions on $\mathcal{M}$, called* contact Hamiltonians*, which are identically zero on $\mathcal{L}$, and $\mathcal{U}$ denotes* *the space of inputs functions u_j. The dynamics is then given by the differential equation:*

$$\frac{d}{dt}(x^0, x, p) = X_{K_0} + \sum_{j=1}^{m} u_j \, X_{K_j} \, . \tag{1}$$

This system may be interpreted in the context of physical system' modelling as follows. Firstly the differential equation (1) defines *a control contact system* in a very similar way to control Hamiltonian systems [17] [18]. It is defined by an internal contact Hamiltonian K_0 generating the drift dynamics X_{K_0} and by interaction contact Hamiltonians K_j defining the external action on the system by the control vector fields X_{K_j}. It is interesting to note that, for physical systems, *the contact Hamiltonians have the dimension of power* and that they are defined by the *law of fluxes* of the system (heat conduction, chemical reaction kinetics or diffusion).

However the system is also defined by a second objet: the Legendre submanifold $\mathcal{L}$ which represents the Thermodynamic properties of the system. Practically this Legendre submanifold is generated by a potential energy function (for instance, the internal energy or free energy of a thermodynamic system, the kinetic and potential energy of a mechanical system). The contact Hamiltonians have to satisfy the *compatibility conditions*, *i.e.* $K_{i|\mathcal{L}} \equiv 0$, which are essential as they express the first principle of Thermodynamics. In other words the conservative control contact system leaves invariant the Legendre submanifold. More precisely these conditions follow from the following result [16].

Theorem 1. *Let $(\mathcal{M}, \mathcal{E})$ be a strictly contact manifold and denote θ its contact form. Let $\mathcal{L}$ denote a Legendre submanifold. Then X_f is tangent to $\mathcal{L}$ if and only if f is identically zero on $\mathcal{L}$.*

Finally note that actually only the restriction of the conservative contact system to the Legendre submanifold (where the first principle is satisfied) is relevant for the description of the dynamics of the system.

Example 1. (Lift of a dissipative Hamiltonian system) This example has been treated in details in [6]. Consider an autonomous *dissipative* Hamiltonian system defined on $\mathcal{N}$ by the equation

$$\dot{x} = (J(x) - D(x)) \, \frac{\partial H_0}{\partial x}(x) \, , \tag{2}$$

where D is the symmetric positive definite matrix of friction. Notice that the tensor $J - D$ defines a Leibniz bracket [15]. It has been shown that it may be embedded into a contact vector field considering:

- the extended base manifold $\mathcal{N}_e = \mathbb{R} \times \mathcal{N}$, and its associated *extended* thermodynamic phase space $\mathcal{T}_e = \mathbb{R} \times T^*\mathcal{N}_e \ni (x^0, x, S, p, p_S)$
- the Legendre submanifold generated by H_e: $H_e(x, S) = H_0(x) + T_0 S$
- the contact Hamiltonian function

$$K_e = -\langle p, \dot{x} \rangle + \frac{p_S}{T_0} \frac{\partial H_0}{\partial x}^t D(x) \frac{\partial H_0}{\partial x} \, . \tag{3}$$

3 Interconnection of Port Contact Systems

In this section, the definition of port outputs conjugated to the control inputs is recalled [7] [6]. The composition of port contact systems is then defined using a power continuous interconnection structure which is not necessarily a Dirac structure, generalizing hence the result in [7]. This latter result is illustrated on the example of a gas under in a cylinder under a piston undergoing some irreversible processes.

3.1 Port Contact Systems and Losslessness

Consider a differentiable real-valued function f on $\mathcal{M}$ and its variation with respect to a conservative control contact system (of definition 1). A straightforward calculation, given below in canonical coordinates[1], leads to the following balance equation :

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \sum_{j=1}^{m} y_f^j + s_f \,, \tag{4}$$

where y_f^j denotes the f-*conjugated output variable* associated with the input u_j:

$$y_f^j = \{K_j, f\} + f\frac{\partial K_j}{\partial x^0} \,, \tag{5}$$

and s_f denotes the *source term* defined by:

$$s_f = \{K_0, f\} + f\frac{\partial K_0}{\partial x^0} \,. \tag{6}$$

For a conserved quantity, the source term is expected to be zero. However, as has been shown in [7] there is no reason to require it on the entire state space but rather only on the Legendre submanifold. This lead to the following definition of a conserved quantity.

Definition 2. *A* conserved quantity *of a conservative control contact system is a real-valued function f defined on $\mathcal{M}$ such that*

$$s_{f|\mathcal{L}} = 0 \,. \tag{7}$$

Definition 3 ([7]). *A* port contact system *is a control contact system with the additional condition that there exists a generating function U of a Legendre submanifold that is a conserved quantity, completed with the U-conjugated output y_U^j defined as in (5).*

Example 2. [5] Consider a port Hamiltonian system defined on a manifold $\mathcal{N}$ endowed with the pseudo-Poisson tensor Λ, the Hamiltonian $H_0(x) \in C^\infty(\mathcal{N})$, an

[1] Recall that the *Jacobi bracket* $\{\cdot,\cdot\}$ of functions on $\mathcal{M}$ is defined as $\{f, g\} = i([X_f, X_g])\theta$, where θ is the contact form defining $\mathcal{E}$, and $[\cdot,\cdot]$ denotes the Lie bracket.

input vector $u(t) = (u_1, \dots, u_m)^T$ function of t, m input vector fields $g_1, \dots, g_m$ on $\mathcal{N}$, and the equations :

$$\begin{cases} \dot{x} = \Lambda^{\#}(\mathrm{d}_x H_0(x)) + \sum_{i=1}^{m} u_i(t)\, g_i(x) \\ {y_p}^j = L_{g_j}.H_0(x) \end{cases} \tag{8}$$

Its lift on the thermodynamic phase space $\mathcal{T} = \mathbb{R} \times T^*\mathcal{N}$ with canonical coordinates (ϵ, x, p), is a port contact system with internal contact Hamiltonian $K_0 = \Lambda(p, dH_0)$, the internal contact interaction contact Hamiltonian are $K_j = \langle \mathrm{d}H_0 - p, g_j \rangle$. The port conjugated output variables defined in (5) becomes

$$y^j_{H_0} = \frac{\partial H_0}{\partial x}^T g_j = L_{g_j} H_0 \ . \tag{9}$$

It is remarkable that they correspond precisely to the outputs called *port outputs* defined in (8).

3.2 Interconnection of Two Port Contact Systems

In this section we consider the interconnection or composition of port contact systems.

Consider now two differential manifolds $\mathcal{N}_i$ of dimension n_i with coordinates $x_i = (x_i^1, \dots, x_i^{n_i})$, for $i = 1, 2$. Each 1-jet space $\mathcal{T}^i$ over $\mathcal{N}_i$ is endowed with a canonical contact structure whose contact form is denoted by θ_i. We now construct the composed state space in the same way. Denote by $\mathcal{N}$ the whole product base space $\mathcal{N}_1 \times \mathcal{N}_2$. Then, the 1-jet bundle $\mathcal{T}$ over $\mathcal{N}$ is also endowed with a canonical contact form θ whose local expression is

$$\theta = \mathrm{d}x^0 - \sum_{j=1}^{n_1+n_2} p_j \, \mathrm{d}x^j , \tag{10}$$

where $x^j = x_1^j$ and $p_j = p_j^1$ if $1 \leq j \leq n_1$, else $x^j = x_2^{j-n_1}$ and $p_j = p^2_{j-n_1}$ if $n_1 + 1 \leq j \leq n_1 + n_2$.

According to definition 3, consider two port contact systems $(\mathcal{N}_i, U_i, K_j^i)$ on $\mathcal{T}^i$ with contact Hamiltonian K_i defined as

$$K_i = K_0^i + \sum_{j=1}^{n_i} u_j^i \, K_j^i , \tag{11}$$

satisfying the invariance condition with respect to the conserved quantity U_i, $i = 1, 2$. We define the new (conserved) generating function U of the Legendre submanifold of the composed state space as $U_1 + U_2$.

Denote by m the number of input variables involved in the interconnection ($m \leq min(m_1, m_2)$). Without loss of generality we may suppose that the first m variables are involved in the interconnection. Denote $u = (u_j^1, u_j^2)$ and $y = (y_1^j, y_2^j)$, $j = 1, \dots, m$.

Proposition 1. *The composition of two port contact systems* $(\mathcal{N}_i, U_i, K_j^i)_{i=1,2}$ *with respect to a* power continuous interconnection relation

$$u = \Phi(y) \quad \textit{together with} \quad \Phi(y)^T y = 0\,, \tag{12}$$

is the port contact system on the Thermodynamical Phase Space $\mathbb{R} \times T^* (\mathcal{N}_1 \times \mathcal{N}_2)$ *defined with respect to the Legendre* $\mathcal{L}_{U_1+U_2}$ *and the contact Hamiltonian* $K(x, p, \Phi(y))$*, where* $K(x, p, u^1, u^2) = K_1 + K_2$*.*

It is obvious to see that the invariance condition is satisfied by K on $\mathcal{L}_U$. We now show that U is a conserved quantity of the interconnected system thus obtained, when restricted to the Legendre submanifold $\mathcal{L}_U$. Indeed, let us compute its time-derivative

$$\frac{\mathrm{d}U}{\mathrm{d}t}_{|\mathcal{L}_U} = \sum_{j=1}^{m} \left[u_1^j \{K_j^1, U_1\} + u_2^j \{K_j^2, U_2\} \right]_{|\mathcal{L}_U}, \tag{13}$$

which is zero by (12) and (5).

The interconnection defined in the proposition 1, strictly generalize the interconnections defined by Dirac structures [20] [21]. Indeed the map Φ in equation (12) defines a power continuous relation but is not necessarily linear hence does not define a vector bundle. As an illustration of this feature we shall present in the next paragraph an example of such a non-linear power continuous interconnection.

3.3 A Gas in a Cylinder Under a Piston

In this paragraph will shall consider a system composed of a gas in a cylinder closed by a piston subject to the gravity. In a first instance, we shall consider that this system undergoes *reversible* transformations. In this case the interconnection structure defining the interaction between the gas and the piston is defined by a Dirac structure (as has been shown in [10]). In a second instance, we shall consider that the system undergoes some *irreversible* transformations due to mechanical friction or viscosity of the gas. In this case the interconnection structure is defined by some map Φ which is non-linear.

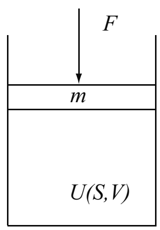

Fig. 1. A gas under a piston

Reversible Transformations Gas Under a Piston

This system may be decomposed in two elementary subsystems, namely the ideal gas undergoing some mechanical work and a mass (the piston) submitted to external forces.

The dynamics of the ideal gas undergoing some mechanical work may be defined as a conservative contact system defined on the Thermodynamic Phase Space $\mathcal{T}_{gas} = \mathbb{R} \times \mathbb{R}^6 \ni \left\{x^0, x^j, p_j\right\}$ where x^i denotes the extensive variables and p_i the conjugated intensive variables. Its thermodynamic properties are given by the Legendre submanifold $\mathcal{L}_U$ generated by the internal energy. As the gas is considered to be in equilibrium in the control volume, the drift dynamics is of course zero. And the external mechanical work provided by an external pressure P^e and variation of volume f_V^e leads to the interaction contact Hamiltonian:

$$K^i_{gas} = (p_2 - P)\ f_V^e\,, \tag{14}$$

with the U-conjugated port output $y_U = -P$.

The dynamics of the piston is given as the lift of a standard Hamiltonian systems on the associated Thermodynamic Phase Space following [5]:

$$\mathcal{T}_{mec} = \mathbb{R} \times \mathbb{R}^4 \ni \left\{x^0, x^{pot}, x^{kin}, p_{pot}, p_{kin}\right\}\,, \tag{15}$$

where x^{pot} denotes the altitude of the piston and x^{kin} its kinetic momentum. The Legendre submanifold $\mathcal{L}_{mec}$ is generated by the total mechanical energy: $H_0 = \frac{1}{2m}\, {x^{kin}}^2 + mgx^{pot}$, which defines the two intensive variables, the velocity v of the piston and the gravity force F:

$$\left.\begin{array}{l} p_{pot}|_{\mathcal{L}_{mec}} = F = mg \\ p_{kin}|_{\mathcal{L}_{mec}} = v = \frac{x^{kin}}{m} \end{array}\right\}\,. \tag{16}$$

The drift dynamics of the piston may be represented by a conservative contact system with internal contact Hamiltonian

$$K^0_{mec} = -\left(p_{pot},\, p_{kin}\right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} F \\ v \end{pmatrix}\,. \tag{17}$$

Associated with the external force F^e exerted on the piston is the interaction contact Hamiltonian:

$$K^i_{mec} = (p_{kin} - v)\ F^e\,, \tag{18}$$

and the H_0-conjugated output: $y_{H_0} = v$, *i.e.* the velocity of the piston. In this first case, following [10], we consider that the composed system is *reversible* and the interconnection is given by the linear relation :

$$\begin{pmatrix} f_V^e \\ F^e \end{pmatrix} = \begin{pmatrix} 0 & A \\ -A & 0 \end{pmatrix} \begin{pmatrix} (-P) \\ v \end{pmatrix}\,, \tag{19}$$

where A denotes the area of the piston and defining a Dirac structure on the port variables $f_V^e, F^e, (-P), v$.

The dynamics of the gas with piston is then given as the composed conservative contact system on the composed Thermodynamic Phase Space:

$$\begin{aligned}\mathcal{T}_{GP} &= \mathbb{R} \times \mathbb{R}^{10} \\ &\ni \left\{x^0, x^j, p_j, x^{pot}, x^{kin}, p_{pot}, p_{kin}\right\}_{j=1,..,3}\,,\end{aligned} \tag{20}$$

with Legendre submanifold generated by the sum of the internal energy and the mechanical energy $U(S,V,N) + H_0(x^{pot}, x^{kin})$ with contact Hamiltonian:

$$K_{tot} = K^0_{mec} + (p_2 - P)\ Av + (p_{kin} - v)\ AP \tag{21}$$

It is interesting to notice that the x-component of the contact field $X_{K_{tot}}$ restricted to $\mathcal{L}_{U+H_0}$ is precisely the port Hamiltonian system proposed in [10].

Irreversible Transformations

In this second case we shall assume that there is some mechanical friction and that the *lost mechanical energy is converted entirely into a heat flow in the gas.*

Firstly the thermodynamical model of the gas, $\mathcal{L}_U$, is remained unchanged. But its dynamics has to be changed as follows, as now there might be a heat flow induced by the mechanical losses. Denoting by $f^e_S = \frac{Q_e}{T}$ the flow of entropy associated with the external heat flow Q_e, there has to be *an additional interaction contact Hamiltonian* to consider:

$$K_S = (p_1 - T)\ f^e_S\,, \tag{22}$$

with its conjugate port output $y^s_U = T$.

Consider that the mechanical losses come from a viscous friction with coefficient ν. The friction losses will be generated by an additional interaction Hamiltonian associated with the dissipative force F^d:

$$K^d_{mec} = (p_{kin} - v)\ F^d\,, \tag{23}$$

with conjugated output v.

The dissipation losses will be taken into account as *an additional interconnection relation* associated with the transformation of part of the mechanical energy in heat. Firstly the friction force is defined and secondly the power continuity of the interconnection defined as follows :

$$(F^d, f^e_S) = \Phi(v,T) = (\nu v, (-v/T)\nu v) \tag{24}$$

An essential feature of this power continuous interconnection is, that it is not a Dirac structure on the port variables (F^d, f^e_S, v, T) as the map Φ in (24) is nonlinear.

The *composed system piston and gas* is defined on $\mathcal{T}_{GP}$, defined in (20), with the canonical contact structure. The properties of the systems are defined precisely as for the reversible case by the Legendre submanifold $\mathcal{L}_{U+H_0}$. The contact Hamiltonian is obtained according to the proposition 1

$$K_{irr} = K_{tot} + \big(p_{kin} - (p_1/T)v\big)\nu v\,. \tag{25}$$

It is immediately seen that the contact Hamiltonian satisfies the invariance condition $K_{irr}|_{\mathcal{L}_{U+H_0}} = 0$. Note that an analogous expression hold if one consider that the gas undergoes some irreversible transformation due to its viscosity. The dynamics restricted to the the Legendre submanifold and projected on the extensive coordinates is:

$$\left.\begin{array}{l}
\left.\frac{dx^S}{dt}\right|_{\mathcal{L}} = \frac{dS}{dt} = -\frac{\partial K_{tot}}{\partial p_S} = \quad \frac{1}{T}\nu v^2 \\
\left.\frac{dx^V}{dt}\right|_{\mathcal{L}} = \frac{dV}{dt} = -\frac{\partial K_{tot}}{\partial p_V} = \quad A\,v \\
\left.\frac{dx^N}{dt}\right|_{\mathcal{L}} = \frac{dN}{dt} = -\frac{\partial K_{tot}}{\partial p_N} = \quad 0 \\
\left.\frac{dx^{pot}}{dt}\right|_{\mathcal{L}} = \frac{dz}{dt} = \quad -\frac{\partial K_{tot}}{\partial p_{pot}} \quad v \\
\left.\frac{dx^{kin}}{dt}\right|_{\mathcal{L}} = \frac{d\pi}{dt} = -\frac{\partial K_{tot}}{\partial p_{kin}} = -F + A\,P = -mg + AP
\end{array}\right\} \tag{26}$$

It may be noted that this formulation of an irreversible transformation of a gas-cylinder system encompasses (by setting $\nu = 0$) the formulation of reversible transformation using a port Hamiltonian system defined on a Dirac structures proposed [10].

4 Conclusion

In this paper we have suggested a definition of power continuous interconnection of conservative contact systems which strictly generalizes the Dirac structures which define the interconnection of port Hamiltonian systems [21]. However the power continuity properties of the interconnection allow to compose conservative contact systems. We have illustrated this in details on the example of a gas in a cylinder and submitted to mechanical work by a piston. On this example we have considered firstly the reversible case, according to the example in [10], and secondly the irreversible by considering some mechanical friction and the entropy balance associated with it.

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5 Appendix: Reminder on Contact Geometry

In this section we shall briefly recall the basic concepts of contact geometry (following [11] and [3]).

Let $\mathcal{M}$ be an $2n+1$-dimensional, connected, differentiable manifold of class C^∞.

Definition 4 ([11]). *A Pfaffian equation on $\mathcal{M}$ is a vector subbundle $\mathcal{E}$ of rank 1 of $T^*\mathcal{M}$. The pair $(\mathcal{M}, \mathcal{E})$ is a* strictly contact structure *if there exists a form θ of constant class $2n+1$, called* contact form*, that determines $\mathcal{E}$.*

Using Darboux's theorem, one shows the existence of canonical coordinates $(x^0, x^1, \ldots, x^n, p_1, \ldots, p_n)$ in a neighborhood V of any $x \in \mathcal{M}$ such that

$$\theta_{|V} = \mathrm{d}x^0 - p_i \mathrm{d}x^i. \tag{27}$$

Definition 5 ([11]). *A* Legendre submanifold *of a $(2n+1)$-dimensional contact manifold $(\mathcal{M}, \mathcal{E})$ is an n-dimensional submanifold $\mathcal{L}$ of $\mathcal{M}$ that is an integral manifold of $\mathcal{E}$.*

Legendre submanifolds are locally generated by some generating function.

Theorem 2 ([2]). *For a given set of canonical coordinates and any partition $I \cup J$ of the set of indices $\{1, \ldots, n\}$ and for any differentiable function $F(x^I, p_J)$ of n variables, $i \in I, j \in J$, the formulas*

$$x^0 = F - p_J \frac{\partial F}{\partial p_J}, \quad x^J = -\frac{\partial F}{\partial p_J}, \quad p_I = \frac{\partial F}{\partial x^I} \tag{28}$$

define a Legendre submanifold of $\mathbb{R}^{2n+1}$ denoted $\mathcal{L}_F$. Conversely, every Legendre submanifold of $\mathbb{R}^{2n+1}$ is defined in a neighborhood of every point by these formulas, for at least one of the 2^n possible choices of the subset I.

Finally we shall recall the definition of the class of vector fields, called contact vector fields, which preserve the contact structure and may be characterized using the following result.

Proposition 2 ([11]). *A vector field X on $(\mathcal{M}, \mathcal{E})$ is an* contact vector field *if and only if there exists a differentiable function ρ such that*

$$\mathcal{L}(X)\,\theta = \rho\,\theta, \tag{29}$$

where $\mathcal{L}(X)$ denotes the Lie derivative with respect to the vector field X. When ρ vanishes, X is an infinitesimal automorphism of the contact structure.

The set of contact vector fields forms a Lie subalgebra of the Lie algebra of vector fields on $\mathcal{M}$.

Analogously to the case of Hamiltonian vector fields, one may associate some generating function to the contact vector fields. Actually there exists an isomorphism Φ between contact vector fields and differentiable function on $\mathcal{M}$ which associate to a contact vector field X a function called *contact Hamiltonian* and defined by :

$$\Phi(X) = i(X)\theta, \tag{30}$$

where $i(X)$ denotes the contraction of a form by the vector field X. In the sequel we shall denote the contact vector field associated with a function f by :

$$X_f = \Phi^{-1}(f). \tag{31}$$

The contact vector field X_f may be expressed in canonical coordinates in terms of the generating function, as follows :

$$\begin{aligned} X_f = &\left(f - \sum_{k=1}^{n} p_k \frac{\partial f}{\partial p_k} \right) \frac{\partial}{\partial x^0} + \frac{\partial f}{\partial x^0} \left(\sum_{k=1}^{n} p_k \frac{\partial}{\partial p_k} \right) \\ &+ \sum_{k=1}^{n} \left(\frac{\partial f}{\partial x^k} \frac{\partial}{\partial p_k} - \frac{\partial f}{\partial p_k} \frac{\partial}{\partial x^k} \right). \end{aligned} \tag{32}$$

Furthermore the isomorphism Φ transports the Lie algebra structure on the differentiable function on $\mathcal{M}$ and defines the following bracket that we shall use in the sequel :

$$\{f, g\} = i([X_f, X_g])\theta, \tag{33}$$

whose expression in canonical coordinates is given by

$$\begin{aligned} \{f, g\} = &\sum_{k=1}^{n} \left(\frac{\partial f}{\partial x^k} \frac{\partial g}{\partial p_k} - \frac{\partial g}{\partial x^k} \frac{\partial f}{\partial p_k} \right) \\ &+ \left(f - \sum_{k=1}^{n} p_k \frac{\partial f}{\partial p_k} \right) \frac{\partial g}{\partial x^0} - \left(g - \sum_{k=1}^{n} p_k \frac{\partial g}{\partial p_k} \right) \frac{\partial f}{\partial x^0} . \end{aligned} \tag{34}$$